Roll-Modulated Lifting Entry Optimization

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The equations of lifting entry are examined for fixed angle-of-attack vehicular motion with path control via roll modulation of lift. A complication arising with this is nonconvexity of the hodograph figure, which makes the application of standard variational techniques inadvisable unless the problem is first relaxed, i.e., a related problem is defined with a hodograph figure that is the convex hull of the original. This leads to a new system in new variables that is apparently innocuous in its simplicity; the linear elements of the convex hull, however, are associated with singular extremal subarcs and their attendant difficulties. The singular extremal for minimum-heating symmetric flight with final time and downrange open is simple. Two order-reduction approximations are considered, which may include intervals of two-dimensional motion as subarcs. One of these approximations relegates turning to initial and terminal boundary-layer maneuvers; the other is analogous to the aircraft energy-maneuvering model. Some computations for a space shuttle orbiter configuration are presented.

Nomenclature

D = drag
E = specific energy
g_o = acceleration of gravity
H = variational Hamiltonian
L = lift
Q = total heat load
$\dot{Q}$ = heat rate
r = radius
r_o = radius of the Earth
V = velocity
W = weight
γ = flight path angle to horizontal
Λ = longitude
λ = Lagrange multiplier
μ = bank angle
ζ = relaxation interpolation variable
σ = relaxation control variable
ϕ = latitude
χ = heading angle to south

State Equations

WITH r radius, γ path angle to horizontal, $E \equiv (V^2/2g_o) - (r_o^2/r)$ specific energy, χ heading angle to south, ϕ latitude, Λ longitude, and μ bank angle, the equations of state are

$$\dot{r} = V \sin\gamma \quad (1)$$

$$\dot{E} = -DV/W \quad (2)$$

$$\dot{\gamma} = (g_o L \cos\mu/WV) - (g_o r_o^2/Vr^2)\cos\gamma + (V/r)\cos\gamma \quad (3)$$

$$\dot{\chi} = (g_o L \sin\mu/WV\cos\gamma) - (V/r)\cos\gamma\sin\chi\tan\phi \quad (4)$$

$$\dot{\phi} = -(V/r)\cos\chi\cos\gamma \quad (5)$$

$$\dot{\Lambda} = (V\sin\chi\cos\gamma/r\cos\phi) \quad (6)$$

$$\dot{Q} = \dot{Q}(E, r) \quad (7)$$

The first six equations are particle-dynamics equations of motion for coordinated maneuvering (zero side-force). The last equation is the total heating integral Q in differential form. Lift L and drag D are functions of E and r only; angle of attack is assumed constant. (If trim were to vary with the Mach number, the angle of attack would itself be a function of E and r.) Inequality constraints on dynamic pressure, normal load factor, and local temperatures are in the problem statement.

Roll Modulation

Entry at essentially constant angle of attack has been employed for such vehicles as the Apollo Command Module, with consequent simplification of longitudinal control. The desired vertical component of lift and a desired average out-of-plane component are obtained by bank reversals, square-wave fashion. In the particle-dynamics model, this includes the theoretical possibility of "chattering," since rigid-body rolling dynamics have been neglected. There is design interest in roll modulation for advanced vehicles such as the Earth-orbital shuttle, even though a longitudinal control system will be featured, since design compromises may force a narrow range of trim angle of attack. Thus, constant angle-of-attack operation is of interest as a limiting case for the shuttle entry problem.

Control Relaxation

In the version of the problem with angle of attack controllable within bounds, the figure in hodograph space ($\dot{E}$, $\dot{\gamma}$, $\dot{\chi}$) that is traced out by varying the controls α and μ over their complete range (Contensou's "Domain of Maneuverability")[1] is not convex. Operation at points within the figure, which is a paraboloid for lift linear and drag quadratic in α, can be approximated by chattering control operation, square-wave fashion, but cannot actually be attained with piecewise continuous controls. In such circumstances, it is usual to consider instead a related problem with different control variables that attain the convex hull of the hodograph figure; this is the "relaxed" problem.[1,2] The relaxation for the variable angle-of-attack case is sketched in Ref. 3. In the present case of fixed angle of attack, the figure is an ellipse. Relaxation makes the disk within this ellipse attainable.

Relaxation may be accomplished for a general state system of the form

$$\dot{x} = f(x, u, t) \quad (8)$$

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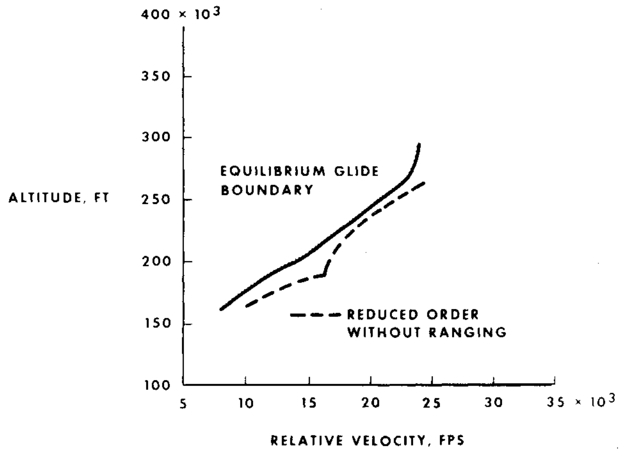

Fig. 1 Minimum-heating trajectory in altitude/velocity chart.

by replacing the system by

$$\dot{x} = f(x, u_1, t) + \zeta[f(x, u_2, t) - f(x, u_1, t)] \quad (9)$$

in which the right members are linearly interpolated between values for control u_1 and control u_2. Here ζ, $0 \leqq \zeta \leqq 1$, is an interpolation parameter. The control variables of the relaxed system are the vectors u_1 and u_2 and the scalar ζ. In the present application, the desired goal of attaining the interior of the ellipse can be accomplished with fewer variables, namely by introducing an additional control variable σ, $0 \leqq \sigma \leqq 1$, multiplicative on L in the $\dot{\gamma}$ and $\dot{\chi}$ state equations

$$\dot{\gamma} = (g_o L\sigma \cos\mu / WV) - (g_o r_o{}^2/Vr^2)\cos\gamma + (V/r)\cos\gamma \quad (3a)$$

$$\dot{\chi} = (g_o L\sigma \sin\mu / WV\cos\gamma) - (V/r)\cos\gamma \sin\chi \tan\phi \quad (4a)$$

Singular Arcs of the Relaxed Problem

The appearance of the control variable σ linearly in the right members of the state equations indicates the possibility of singular arcs in the solution of optimal entry control problems. This possibility may be investigated by formation of the usual Hamiltonian H, setting $\partial H/\partial\sigma = 0$, and pursuing the consequences.

$$\partial H/\partial\sigma = (g_o L/WV)[\lambda_\gamma \cos\mu + \lambda_\chi (\sin\mu/\cos\gamma)] = 0 \quad (10)$$

$$\partial H/\partial\mu = (g_o L\sigma/WV)[-\lambda_\gamma \sin\mu + \lambda_\chi (\cos\mu/\cos\gamma)] = 0 \quad (11)$$

Left members of Eqs. (10) and (11) must vanish independently. Since these are linearly independent, it follows that both λ_γ and λ_χ are zero along the arc.

The system is already in the canonical form of Ref. 4; thus the variables γ and χ are control-like along singular arcs. A similar result could have been obtained by noting that $\sigma \sin \mu$ and $\sigma \cos \mu$ could be taken as new control variables in the neighborhood of a singular arc for $0 < \sigma < 1$. Desired variations in γ and χ can be realized by varying these, as long as the magnitude of the desired variations is sufficiently small as not to encounter saturation of the σ bounds.

With γ and χ regarded as controls, the problem simplifies to flight in the plane of a great circle. Without loss of generality, take $\mu = \phi = 0$, and $\chi = \pi/2$ for study of this two-dimensional motion, and the state equations become

$$\dot{r} = V\sin\gamma \quad (12)$$

$$\dot{E} = -DV/W \quad (13)$$

$$\dot{\Lambda} = V\cos\gamma/r \quad (14)$$

$$\dot{Q} = \dot{Q}(E, r) \quad (15)$$

In the special case of downrange open (final Λ unspecified for initially equatorial flight), the control variable γ enters only Eq. (12) and the variable r becomes control-like along singular arcs as the form with Eq. (12) deleted is again canonical. If final time is open, there is analytical advantage in casting E in the role of independent variable; furthermore, the steady decrease of E makes this interchange feasible for entry applications.

$$dQ/dE = -W\dot{Q}/DV \quad (16)$$

The singular extremal is defined by stationary points of the right member of Eq. (16) regarded as a function of r at various E levels. The generalized Legendre-Clebsch[4] test requires that the stationary value of the right member of Eq. (16) as a function of r be a maximum and $\dot{Q}/DV$ minimum.

Reduced Relaxed Problems

The relaxed problem presents computational difficulties because of singular arcs; approximations are therefore of more than usual interest. Possibilities offered by singular perturbation procedures[5-7] are discussed in the following paragraphs. If nearly symmetric flight were assumed, a singular perturbation approach designating latitude, longitude, and heating as variables of a reduced solution (i.e., solution of a reduced-order approximation problem) would seem attractive. This would relegate turning and altitude transitions to corrective boundary-layer transients near initial and terminal points. Energy is chosen as the independent variable. The reduced problem is of the great-circle type.

The great-circle reduced-order system for the approximation that combines altitude and heading transients takes the form

$$d\Lambda/dE = -W/Dr \quad (17)$$

$$dQ/dE = -W\dot{Q}/DV \quad (18)$$

The order-reduction procedure used is the same one examined and employed in Ref. 6. An upper bound on the control variable r of the reduced problem is furnished by the control bound $\sigma = 1$ of the original problem together with Eq. (3a) and $\dot{\gamma} = \gamma = 0$; a lower bound is provided by state inequalities on panel temperatures and acceleration loads, handled in penalty function approximation in the computations next described. Use of the model given by Eqs. (17) and (18) is limited to problems for which downrange is specified as greater than, or equal to, the downrange-open value; for smaller specified downrange, the singular extremal fails the generalized Legendre-Clebsch test and a zigzag competitor is optimal.

A less drastic approximation using singular perturbations would treat heading as well as latitude, longitude, and heating in a reduced problem. This would idealize only the altitude transients as fast (with respect to energy change) compared to the other transitions. It is the same as aircraft energy approximation.[3,6] Energy approximations have previously been examined for atmospheric entry of a variable angle-of-attack vehicle.[7] No complications arising from the relaxed model are anticipated using this approach. Evidently a solution for the reduced-order fixed angle-of-attack problem consists generally of a turning arc, a great-circle time-open arc, and, if final heading is specified, a terminal turning arc.

Computational Results

Data for a delta-wing space-shuttle orbiter configuration were used for some sample computations with the model of Eqs. (17) and (18). The angle of attack was fixed at 30°. Inequality constraints on normal load factor and numerous panel temperatures were incorporated by using penalty functions.

A minimum of the Hamiltonian consisting of a linear combination of the right members of Eqs. (17) and (18) plus penalties was found by one-dimensional search. With downrange open, the minimum always occurred at the lower bound on altitude furnished by the load factor and temperature constraints (see Fig. 1). With downrange specified at values exceeding the open value, the minimizing altitude was found to be the upper bound value ($\sigma = 1$) during the latter part of the trajectory (see Fig. 2). As range requirements were increased, numerical results indicated the possibility of more than one switch between altitude bounds. The Hamiltonian function for the downrange-specified case of

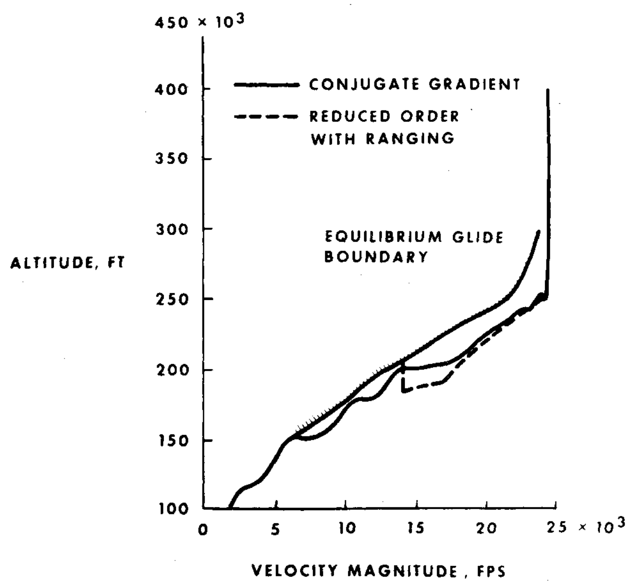

Fig. 2 Minimum-heating downrange-specified trajectories in altitude/velocity chart.

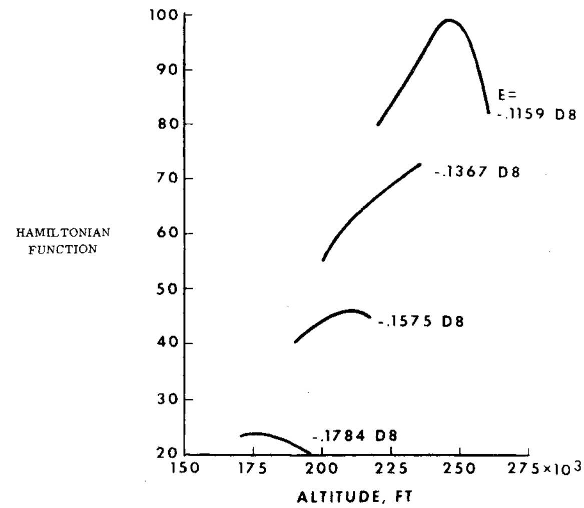

Fig. 3 Hamiltonian vs altitude at several energy levels for downrange-specified case.

Fig. 2 is sketched vs altitude in Fig. 3 for several energy values. The sign of the second derivative H_{rr} would seem to indicate nonconvexity and a need for further relaxation. However, one recalls that r in the role of control variable is not the real thing but the result of an order-reduction approximation amounting to assumed instantaneous vertical dynamics. This implies that weak as well as strong minima should be considered; hence, that transitions determined according to absolute minimum H, as in Fig. 2, are somewhat arbitrary.[6] Boundary-layer transition fairings at discontinuities in r, as in Ref. 6, are needed for consistency in degree of approximation of the control, but they contribute nothing to the performance index in this approximation.

When heating was heavily weighted compared to down-ranging, values of σ were found to be below unity indicating a need for roll modulation in two-dimensional flight. However, solutions with downrange heavily weighted ride the upper bound $\sigma = 1$ at low energies and at near-orbital energies.

Results obtained by a conjugate gradient method that used a particle-dynamics model are shown for comparison. The cross-range was specified at a somewhat challenging value of about 1300 nm. The conjugate gradient formulation did not employ a relaxed model and was unsuitable for nearly symmetric flight cases. It exhibited poor convergence that was, perhaps, attributable to the absence of convexity. Nonetheless, the result of Fig. 2 seems of interest for the qualitative similarity of the altitude history with the great-circle model. This was obtained using as a first guess a trajectory which had been forced to follow the lower bound representing temperature limit approximately. The comparisons suggest that the idealization of early heading and altitude transitions followed by altitude control based mainly on heating and down-ranging may warrant further investigation. A separate treatment of the initial transition as a boundary layer in which altitude, path angle, and heading motions are fast (with respect to E changes) could be carried out along the lines of that for aircraft altitude transitions in Ref. 6.

Conclusion

Attention has been directed to relaxation and its consequences for the fixed angle-of-attack atmospheric entry problem. Two reduced-order approximations for the resulting system of equations have been briefly examined and appear to warrant additional study.

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